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Statistical Treatment of Pellet Dispersion Data for Estimating Range of Firing

When a shotgun is fired, the pellet charge emerges from the muzzle as a single mass and remains so for a couple of feet, after which the pellets begin to disperse. This dispersion increases with the range of firing. The relationship between the size of pellet pattern and the range of firing is routinely employed in forensic science laboratories to estimate the range of firing. The method consists of firing test shots from different distances using the weapon of the crime and ammunition similar to that used in the crime and ascertaining the limits of the distance within which a pattern of the size of the evidence pattern can be obtained. This approximately defines the limits of the range of firing.

The accuracy of the results depends primarily upon the ability of a firearms examiner to obtain the firearm used in the commission of crime and a sufficient quantity of proper test ammunition. The best test ammunition is that belonging to the same lot and batch as the cartridge of the crime. In practice this requirement proves to be rarely satisfied. Most of the time one can, at best, ascertain the type and make of ammunition used in a crime. Only in rare cases is it also possible to know the year and the month of manufacture. Even if all of these data are available it becomes difficult to ensure that the test ammunition had been stored under the same conditions as the crime cartridge. These factors introduce an element of uncertainty into all range determinations. Firearms examiners usually rely upon their experience and the results of experimental tests in setting the limits for the probable range of firing. In practice the range of firing is almost always correlated with the size of pattern, which is obtained by averaging the horizontal and vertical dispersions of test shots (Fig. 1). Even if a firearms examiner is able to obtain the proper test ammunition, he is seriously handicapped by the fact that the quantity of it which is available is usually limited. With this limited ammunition he has to fire test shots from varying distances. To take into account round-to-round variations he has to fire several shots from a particular distance. The experimental data so obtained is then interpreted generally in the light of a firearms examiner's experience. Seldom is an attempt made to analyze the data statistically, which is essential for an objective estimate of the range of firing.

Thus, from a limited body of data in the form of a few test shots from varying distances a firearms examiner tries to assess the limits within which the dispersion of pellets will fall at different distances. Without using statistical methods for this purpose he is not justified in estimating these limits. It is generally accepted that a knowledge of the technique of measurement must form the basis of laboratory work in criminal investigation. A knowledge of the technique of measurement includes recording and treating data with a proper

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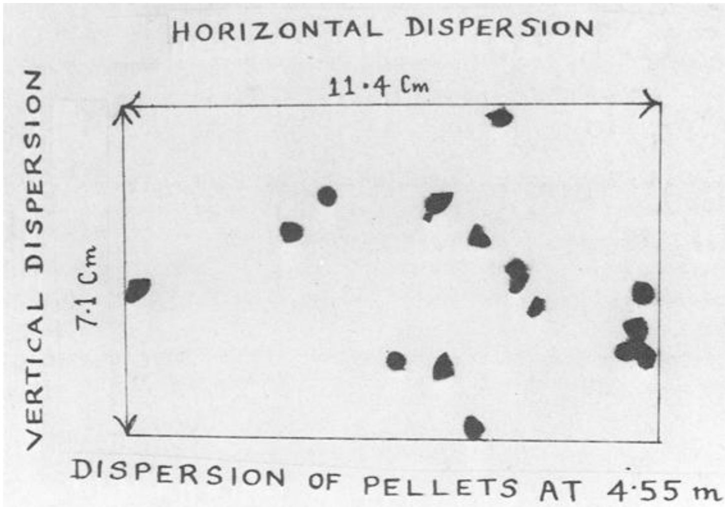

FIG. 1—*A typical pellet pattern obtained by firing a 0.410 musket at a distance of 4.55 m. The size of the pattern was obtained by averaging the horizontal and vertical dispersions as indicated in the figure. Choke = nil. Weight of powder charge = 18 grains of cordite 1; weight of shot charge = 10.2 g; diameter of pellet = 0.46 cm.*

appreciation of the accuracy of the instruments and a method of judging the reliability of results. A correct treatment of data and proper presentation of conclusions can go a long way toward preventing any miscarriage of justice. In this paper, therefore, a standard experimental procedure for estimating the range of firing using a limited quantity of ammunition has been suggested. A procedure for the statistical treatment of the data so obtained has also been outlined to provide the desired objectivity in the estimate.

Experimental Procedure

When one tries to correlate the range of firing with the size of the pattern, he fires several test shots at a target from different distances. If one were to fire ten shots from each of six different ranges of firing, sixty cartridges would be needed. This quantity is much too large from a practical standpoint. The quantity of ammunition required can be much reduced by firing through an array of vertical and parallel thin paper screens fixed at varying distances (Fig. 2) from the muzzle of the gun. It is convenient to keep the consecutive screens at a distance of 0.91 m (3 ft) apart. The screen nearest the muzzle should not be closer than 0.91 m, as at distances less than this the pellets do not show any dispersion. When a firearms examiner encounters an evidence pattern he can, based on his experience, roughly judge the range of firing. In the test, the array of screens may then be hung so that the roughly estimated range of firing falls well within the distances of the screens nearest to and farthest from the muzzle of the gun. Several shots (depending upon the number of test cartridges available) may then be fired, which will enable the firearms examiner to have as many test patterns at each distance as the number of cartridges fired. The sizes of the patterns can then be measured.

Obviously, one can object that in using paper screens the resistance of each sheet of paper (however thin) may progressively affect the size of the pattern and thus cause error. With respect to this, whenever range determinations are made on the basis of the size of pellet pattern a firearms examiner makes certain that the evidence pattern is the entire pattern and not a fragment of a pattern, for estimation of range of firing from a fragment of a pattern may be wholly unreliable. Inasmuch as the human body presents a small target, usually at ranges beyond 9.1 m (30 ft) the entire pellet pattern is not recorded. Thus when one is dealing with the whole pattern one is not dealing with a long distance of firing. However, if the pattern is on a wall or some similar surface it may be recorded in its entirety, even at long distances. At distances close to the muzzle the velocity of pellets is high, and if the paper screens are few and thin there is no significant effect upon the dispersion of pellets.

This was demonstrated by experimental firing conducted with a 0.410 smooth bore musket (true cylinder) firing 0.410, MK1, K.F. cartridges filled with 18 grains of cordite 1 and 18 pellets of lead each measuring 0.46 cm in diameter. Firing (ten shots) was conducted through thin paper screens (0.013 cm thick) placed at distances ranging from 0.91 m (3 ft) to 5.46 m (18 ft) from the muzzle of the musket. The screens were hung 0.91 m (3 ft) apart. The sizes of the patterns obiained on the screen placed at a distance of 4.55 m from the muzzle were noted and are given in column 2 of Table 1. The intervening screens were then removed and again ten shots were fired directly onto the screen placed at a distance of 4.55 m. The sizes of the patterns obtained are given in column 3 of Table 1.

TABLE 1—*Pattern sizes: firings with and without intervening paper screens. Range of firing = 4.55 m (15 ft).*

Shot No.	Size of Pattern with Four Intervening Paper Screens, cm	Size of Pattern without Intervening Screens, cm
1	11.0	7.9
2	9.2	14.2
3	13.6	11.9
4	11.1	11.8
5	8.5	11.7
6	15.5	11.4
7	15.3	6.7
8	13.7	15.6
9	15.0	11.5
10	13.9	10.0
$\bar{x}$	12.68	11.27
s	2.5495	2.6268

The arithmetic means and unbiased estimates of the standard deviations are given at the bottom of the table. If $\bar{x}_1$ and $\bar{x}_2$ are the arithmetic means of the observations given in columns 2 and 3 of Table 1 and s_1 and s_2 are the unbiased estimates of standard deviations, the t test may be applied to test the significance of the difference of means $\bar{x}_1$ and $\bar{x}_2$. For this purpose t will be defined as

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\dfrac{\sum (x_1 - \bar{x}_1)^2 + \sum (x_2 - \bar{x}_2)^2}{n_1 + n_2 - 2}}\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}}$$

for $(n_1 + n_2 - 2)$ degrees of freedom, where $n_1 = n_2 = 10$ = number of shots fired and, hence, the number of observations taken on the pattern size. On substituting for the various quantities, we get

$$t = \frac{(12.68 - 11.27)\sqrt{5}}{\sqrt{\dfrac{58.4760 + 62.1210}{18}}} = 1.21$$

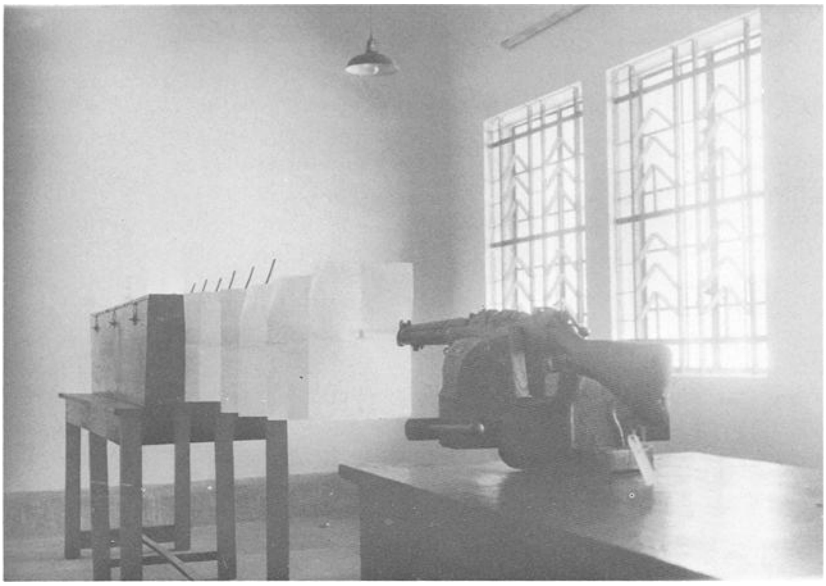

FIG. 2—*Experimental arrangement showing the 0.410 musket and paper screens (0.013 cm thick) in front of bullet recovery box.*

which is insignificant at a 5 percent level. Thus the difference between the means is insignificant at 4.55 m and, as such, it can also be assumed to be insignificant at lower ranges of firing. In a practical situation one can test this significance at the distance at which the farthest screen is placed and, if this is insignificant, it can be assumed to be so at smaller distances also.

Statistical Treatment of Pellet Dispersion Data

The statistical treatment of pellet dispersion data can be undertaken by two methods: (1) distribution-free method and (2) method used for normal populations. To illustrate the various methods with actual data, firing was conducted with a 0.410 musket through paper screens (0.013 cm thick) placed at distances ranging from 0.91 m (3 ft) to 5.46 m (18 ft) from the muzzle of the musket. Ten shots were fired from a musket held in a vise. Thus ten patterns were available at each of six distances. The sizes of the patterns are given in Table 2. The arithmetic mean and unbiased estimate of the standard deviation calculated for each distance are given at the bottom of the table. From the data available in Table 2, the lower and the upper limits for the size of the pattern at each distance (0.91 m to 5.46 m) must be specified so that the sizes of a certain percentage of patterns fall with certainty within these limits.

The distribution-free method is adopted when it is not possible to justify the assumption of a normal distribution. If one is dealing with a statistical variable that can be described by a continuous distribution, one can determine the confidence limits with the help of the smallest and the largest observations. Thus x_{min} and x_{max}, the smallest and the largest values of the size of pattern observed at various ranges, will become the lower and the upper limits. It is obvious that the confidence in such limits will depend on the number of test shots fired. Distribution-free limits calculated on the basis of data in Table 2 are given in Table 3. The method used for a normal population envisions that (1) all the causes of

TABLE 2—*Pattern sizes at six different distances for ten consecutive firings.*

	Size of Pattern, cm, at Distance of Firing					
Shot No.	0.91 m	1.82 m	2.73 m	3.64 m	4.55 m	5.46 m
1	1.5	3.4	5.4	8.1	11.0	13.9
2	2.5	2.8	4.1	6.4	9.2	11.9
3	1.8	3.5	6.3	9.7	13.6	16.6
4	2.1	3.5	6.0	8.4	11.1	14.4
5	2.1	2.8	4.3	5.8	8.5	10.6
6	2.1	4.7	8.5	11.9	15.5	19.0
7	1.9	4.1	7.0	10.6	15.3	19.5
8	2.0	4.2	6.6	10.2	13.7	15.5
9	2.2	4.3	7.6	11.1	15.0	19.3
10	2.0	4.2	7.1	10.6	13.9	16.2
$\bar{x}$	2.02	3.75	6.29	9.28	12.68	15.69
s	0.2615	0.6481	1.3928	2.0347	2.5495	3.0692

NOTE—The firing was conducted from distances of 3, 6, 9, 15, and 18 ft and the dispersion of pellets was measured in centimeters. To make the distance of firing also conform to the metric system, these were later converted into meters as shown above (3 ft = 0.91 m).

TABLE 3—*Distribution-free confidence limits calculated from data of Table 2.*

	Limits, cm, at Distance of Firing					
	0.91 m	1.82 m	2.73 m	3.64 m	4.55 m	5.46 m
Lower	1.5	2.8	4.1	5.8	8.5	10.6
Upper	2.5	4.7	8.5	11.9	15.5	19.5

variability in the size of the pattern must be detected and eliminated so that whatever variability remains may be considered as random and (2) the statistical population under study is assumed to be normal.

When a firearms examiner performs experimental firing in the laboratory, he tries to approximate the conditions prevailing when the shot of the crime was fired. Thus, he experiments with the crime weapon using ammunition similar to that used in the crime. Under these circumstances all causes of variability except those caused by round-to-round variations are more or less eliminated. The round-to-round variations can be taken to be random. Under these conditions the distributions of populations consisting of the size of the patterns at different distances may also be treated reasonably as normal. Thus, as both of the above conditions are satisfied, one may proceed with the method adopted for normal populations.

The ten observations at each of the six distances (Table 2) form a random sample of size ten from a normal population having some mean and standard deviation. Let the mean and standard deviation of the population of the size of patterns at a particular range of firing be μ and σ, respectively. If μ and σ are known these limits will obviously be $\mu \pm z\sigma$, where z simply depends on the proportion of population which is included within the limits. For example, the limits $\mu \pm 1.645\sigma$ include 90 percent of a normal population with mean μ and standard deviation σ. In criminal investigation a single evidence pattern will be available, the size of which can be measured. Ten patterns have been obtained at each of the various distances ranging from 0.91 m to 5.46 m by firing ten experimental shots. Thus at each distance there are only ten observations from which it is hoped to be able to determine unbiased estimates of the population mean and standard deviation. But one cannot by any means determine the mean and standard deviation of the population of which the ten observations constitute a sample at a particular range of firing. If $\bar{x}$ and s are the mean and standard deviation of the ten observations at a particular distance, it cannot be said that $\bar{x} \pm 1.645s$ will also include 90 percent of the population. The proportion of the population that lies between $\bar{x} \pm ks$ (k being some numerical factor) depends upon how closely $\bar{x}$ and s estimate μ and σ, respectively. Since $\bar{x}$ and s and hence $\bar{x} \pm ks$ are random variables, it is impossible to say with certainty that $\bar{x} \pm ks$ will always contain a specified proportion P of the population. Thus it is impossible to choose k so that the calculated limits will always cover a specified proportion P of population.

This situation, however, is not as hopeless as it seems, because it is possible to determine k so that in many random samples (obtained by firing test shots) from a normal population a certain fraction γ of the intervals $\bar{x} \pm ks$ will contain $100P$ percent or more of the population. Here P is referred to as the coverage and γ as the confidence coefficient. Thus there is 100 γ percent confidence that the limits $\bar{x} \pm ks$ will include at least $100P$ percent of normal population. It is reasonable to expect that the value of k used with $\bar{x}$ and s will be taken large enough. Only then can the probability that $\bar{x} \pm ks$ will contain at least $100P$ percent of the population be made close to 1.

Thus a firearms examiner is faced with a decision: make a broad statement with little risk of error or make a precise statement (namely, narrow range) with greater risk of error. The problem, statistically speaking, becomes that of finding the smallest value of k consistent with a specified confidence coefficient γ, proportion P, and sample size n. The values of factor k such that the probability is 0.95 that at least a proportion P of the distribution will be included between $\bar{x} \pm ks$ calculated from a sample of size n are available in the literature. These values are given in Table 4. The confidence limits $\bar{x} \pm 4.433s$ will determine the limits which include at least 99 percent of the sampled population, and this can be said with 95 percent confidence. The 95 percent confidence limits including at least 75, 90, 95, and 99 percent of the sampled population have been calculated and are given in Table 5.

TABLE 4—*Values of* k *for varying values of* P *and* n.

	k at P values of			
n	0.75	0.90	0.95	0.99
10	1.987	2.836	3.379	4.433
17	1.679	2.400	2.858	3.754
37	1.450	2.073	2.470	3.246
145	1.280	1.829	2.179	2.864

Discussion

At the beginning two problems were posed, namely, to devise an experimental arrangement with the help of which it may be possible to obtain a sufficient number of test patterns with a limited quantity of test ammunition and to outline a procedure for the statistical treatment of pellet dispersion data. The former has been achieved by using a limited number of thin paper screens separated by a fixed distance. The number of screens has to be determined by the firearms examiner after examining the evidence pattern. He has to ensure that the approximate range of firing, as visualized from the evidence pattern on the basis of experience, falls well within the distance of the nearest and the farthest screens. A distance of 0.91 m between consecutive screens, which has been found to be suitable in the present study, can be reduced or increased as long as the separation chosen between any two consecutive screens results in an obvious measurable change in the size of the pattern. The effect of intervening screens has to be examined by firing several shots at the farthest screen without any intervening screens and seeing that the difference in the dispersion of pellets caused by the screens is statistically insignificant. Use of a minimum number of thin paper screens will help in reducing the effect of the screens on the dispersion of pellets. This procedure results in a considerable saving of ammunition and also the valuable time of firearms examiner. Only a few cartridges may be recovered from the suspect; hence, the method of using paper screens is of especial significance, as it enables one to obtain a sufficient number of test patterns with a limited quantity of ammunition, thereby making the estimate of range of firing as reliable as possible under the circumstances.

TABLE 5—*Ninety-five percent confidence limits for varying firing distances and percentages of the sampled population.*

Limits Including at Least		Limits, cm, at Distances of Firing					
		0.91 m	1.82 m	2.73 m	3.64 m	4.55 m	5.46 m
75% of the Sampled	Lower	1.5	2.5	3.5	5.2	7.6	9.6
Population	Upper	2.5	5.0	10.1	13.3	17.7	21.8
90% of the Sampled	Lower	1.3	1.9	2.3	3.5	5.4	7.0
Population	Upper	2.8	5.6	10.2	15.0	19.9	24.4
95% of the Sampled	Lower	1.1	1.6	1.6	2.4	4.1	5.3
Population	Upper	2.9	5.9	11.0	16.1	21.3	26.1
99% of the Sampled	Lower	0.9	0.9	0.1	0.3	1.4	2.1
Population	Upper	3.4	6.6	12.5	18.3	24.0	29.3

The statistical treatment of pellet dispersion data is considered essential to minimize the element of subjectivity. Objective estimates enable a forensic scientist to more thoroughly evaluate the reliability of his results. The distribution-free method of determining confidence limits is rapid and simple. However, the reliability of the results was found to depend upon the number of test patterns available for evaluation. To ascertain whether or not the distribution-free method could yield reliable estimates of range of firing with ten experimental shots (the number adopted in the present study), one of us (Chatterjee) was asked to fire one shot from some distance (between 0.91 and 5.46 m) using the test musket and ammunition belonging to the same lot and batch as the test ammunition. He was also asked to fire another shot from some distance (again between 0.91 and 5.46 m) using the same musket with ammunition of the same make as the test ammunition but not of the same lot and batch. The two patterns so obtained were handed over to M. Jauhari, who was asked to estimate the range of firing using the distribution-free limits given in Table 3. The sizes of these patterns were found to be 5.0 cm and 5.1 cm, respectively, that is almost equal.

Reference to Table 3 shows that the corresponding lower limit firing distance for a pattern of size 5.0 cm will be somewhere between 2.73 and 3.64 m. A linear interpolation determines the more exact figure to be 3.21 m. The upper limit for a pattern size of 5.0 cm falls between 1.82 and 2.73 m and by linear interpolation is 1.89 m. Thus the range of firing lies between 1.89 m and 3.21 m. The actual range of firing was 3.05 m. Thus the estimate of range of firing is correct. Next, the same estimates were made for the other pattern, which was obtained by firing the same musket as used in the experimental tests with ammunition of the same make as the test ammunition but not from the same lot and batch. This is a practical difficulty which every forensic scientist has to face, since frequently he is unable to ascertain the lot and batch of the cartridge of the crime. Using distribution-free limits it was found that the estimate of the range of firing was between 1.91 m and 3.26 m. The actual range of firing in this case was 2.13 m, and hence our estimate is again correct.

As a further test, another pattern was prepared using the same musket and ammunition belonging to the same lot and batch as the test ammunition. The size of this pattern was 9 cm. Using distribution-free limits it was found that this represented a range between 2.86 and 4.76 m. The actual range of firing in this case was 3.05 m. Thus this estimate too is correct. In the previous firing a pattern of size 5 cm has been obtained from 3.05 m and in the latter a pattern 9 cm in size had been obtained from the same distance. This gives an idea of round-to-round variation among cartridges belonging to the same lot and batch. This variation is also reflected in the data given in Table 2. Distribution-free limits calculated by firing ten shots gave reliable estimates of the range of firing in the case under study. The fact that considerable differences in the size of the pattern may result, even when shots are fired from the same distance using ammunition of same lot and batch, and also that almost equal size patterns may be obtained from distances separated by about a meter, should act as a solemn warning to those enthusiasts who sometimes try to make precise estimates of the range of firing with a limited quantity of ammunition and without recourse to statistical methods of analysis. The estimate of range of firing provided by statistical procedures is always in the form of an interval, which ensures presentation of the latitude that is so desirable in opinions expressed in connection with criminal investigation.

The limits shown in Table 5 have been calculated on the assumption of a normal population. In stipulating these limits one is 95 percent confident that at least a certain percentage of the sampled population will fall within them. These limits have been calculated with the formula $\bar{x} \pm ks$ where, as stated earlier, k is a numerical factor. The values of k have been listed in Table 4. Reference to Table 4 shows that for a particular size n of sample the value of k increases with an increase in the proportion P of the population to be included. However, for a particular proportion of population to be included the value of k decreases with the size of the sample. To include at least 99 percent of the sampled population within the limits $\bar{x} \pm ks$, the value of k must be decreased from 4.433 for a sample of size 10 to 2.864 for a sample of size 145. The smaller the value of k for a particular proportion of sampled population to be included, the smaller the interval between $\bar{x} - ks$ and $\bar{x} + ks$. Thus when k is small the estimate of the range of firing is spread over a shorter interval and hence is more precise. If k is large the interval between $\bar{x} - ks$ and $\bar{x} + ks$ will be large and hence the estimate of range of firing will be vague.

Thus, one of the ways of making the estimate more precise will be to increase n, that is, increase the number of test shots. This, however, is not always within the control of a firearms expert because he has to depend upon the availability of proper test ammunition. If a sufficient quantity of test ammunition is available it may be desirable to fire a large number of shots. When a large number of test patterns is available the reliability of distribution-free method also increases considerably. It is the availability of test ammunition that always restricts the value of n, or the number of test patterns obtainable at various distances. In such a case a lower value of k can only be obtained by sacrificing the smallest proportion of the sampled population to be included within the limits. It is seen from Table 4 that for $n = 10$ the value of k is 1.987 when at least 75 percent of the sampled population is included within the limits $\bar{x} \pm ks$. It increases to 4.433 when the smallest proportion included is raised to 99 percent. There is hardly any doubt that one will try to include the highest percentage of the population within the limits. But it must be kept in mind that if this leads to widely separated limits, such limits may not be of any practical interest. The firearms examiner must strike a balance between the length of the limit interval and the certainty provided by including a high percentage of the sampled population within the limits.

This can be illustrated with the help of Table 5, where 95 percent confidence limits have been calculated for the inclusion of at least 75, 90, 95, and 99 percent of the sampled population. For a pattern of size 5.0 cm the ranges of firing distance, in meters, will be 1.82 to 3.53, 1.62 to 4.36, 1.55 to 5.23, and 1.36 to . . . (the upper limit could not be calculated as data beyond 5.46 m were not available) for the inclusion of at least 75, 90, 95, and 99 percent of the sampled population, respectively. The actual range of firing, 3.05 m, is contained in all the intervals. But all of these intervals are wide except for the one calculated by the inclusion of at least 75 percent of the sampled population. It will also be seen that when at least 99 percent of the sample population is included it is not possible to provide the upper limit for the range of firing, since this requires pellet dispersion data beyond 5.46 m. Thus, for all practical purposes, one may be content in specifying 95 percent confidence limits so that at least 75 percent of the sampled population is included within the limits $\bar{x} \pm ks$. The inclusion of at least 99 percent of the sampled population results in a wide interval which may not be of practical interest. By reducing this percentage to 75 it has been possible to make the estimates more precise without incurring a greater risk of error.

Summary

An arrangement for carrying out tests in connection with the estimation of range of firing distance with the help of pellet dispersion data has been described. Two procedures for the statistical treatment of experimental data have been outlined. The statistical treatment of data has been considered essential in eliminating the element of subjectivity from and introducing greater objectivity into the estimates of range of firing.

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